11 How to Balance Quality and Logistics in Food Supply Chains

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Abstract

In this paper we work out a very simple two-echelon inventory model based on a perishable product (milk). The two echelons are: "retail store" and "warehouse". This model gives insight how to balance the cooling costs against the costs related to the reduction of system sojourn time of the product by choosing smaller lot sizes in warehouse and/or in the retail store.
Finding the optimal balance itself has been achieved by minimization of the total relevant cost (set up costs, inventory costs and costs related to cooling the product) of this two-echelon system taking into account the restriction that the number of micro-organisms per ml milk never exceeds a pre-specified threshold. This illustration makes clear that for relevant choices of problem data this balancing is meaningful.

My contribution is dedicated to the memory of my colleague Jo van Nunen who suddenly passed away on May 12, 2010. I have known Jo since the early Seventies. I deeply acknowledge his outstanding achievements in Operations Research, Logistics and Information Technology during many decades.

11.1 Introduction

Often there are two ways (and all relevant combinations) to keep quality of fresh food products in supply chains on desired levels:

1. by spending effort on cooling the products during their stay in the supply chain
2. by spending effort on speeding up the goods flow in the chain by choosing smaller production and transportation lots.

In the framework of research in food supply chains we developed a model that describes a very simple two-echelon system of a perishable product (milk) that is stored in a warehouse and is forwarded, in batches, to the retail store periodically.

For this highly simplified situation this model illustrates how to balance on the one hand the cooling costs in the chain (determination of temperatures in warehouse and retail store) and on the other the reduction of sojourn times in the chain by choosing smaller lot sizes in warehouse and/or in the retail store (set up costs and inventory holding costs).

The reason for this over-simplification is that introduction of new concepts should preferably start with an illustration of how these concepts work in a very simple (possibly oversimplified) situation. It is my belief that in this way the chance that these concepts will eventually be adopted in more realistic settings (this is the ultimate goal!) is increased significantly.

The only purpose of this contribution is to give impetus to research in simultaneous control of temperature and logistics in more realistic chains or networks of perishable products. This contribution is structured as follows. First we discuss the isolated situation of the retailer. Second we discuss the two-echelon situation in which the warehouse and the retailer are considered simultaneously. Third we formulate some conclusions and recommendations.

11.2 Determination of lot size and temperature at the retailer store

Consider a very simple lot sizing situation (see figure 1) with constant demand D and lot size Q.

In figure 1 the development over time of the inventory is described. At points of time where inventory drops to zero a replenishment of *Q* takes place (see Silver et al. 1998).

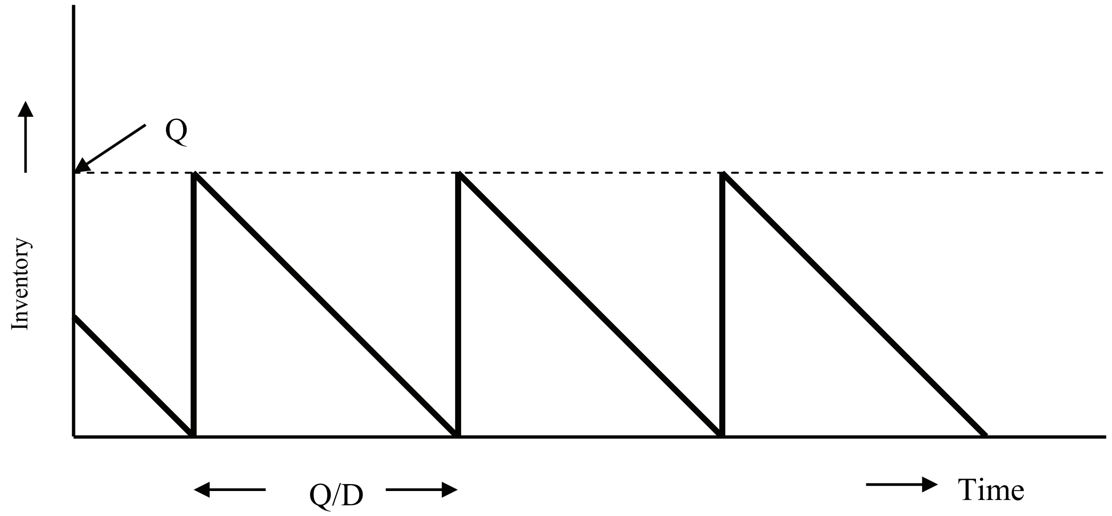

Figure 1: Inventory at the retailer store

The time between the consecutive replenishments from the warehouse amounts to *Q/D* periods.

Let *h* be the inventory cost coefficient and *F* the fixed setup cost. It is very well known that the following cost function has to be minimized in order to obtain the best value for *Q* :

$$(1) \qquad C_{tot}(Q) = \frac{1}{2} h Q + \frac{FD}{Q}$$

By putting $C'_{tot}(Q) = 0$ the so-called Economic Order Quantity (EOQ) is obtained:

$$EOQ = \sqrt{\frac{2FD}{h}}$$

Now assume that the underlying product is perishable (e.g. milk) and its decay depends on time *and* temperature. In the case of milk the process of decay is induced by micro-organisms and their growth is assumed to obey (as an illustration) the following formula:

$$(2) \qquad N_{t+1} = N_t e^{\mu(T)W}$$

where: N_t : = # micro-organisms per ml (time *t*)
W : = sojourn time
$\mu(T)$: = growth coefficient of the micro-organisms
T : = temperature

Other perishables (e.g. meat) require alternative formulae (see Zwietering 1993).

Furthermore, let *G*(*T*) be the cost per period for keeping the product on *T* degrees. Now we can draw up the following optimization model comprising inventory costs, set up costs and costs related to cooling combined with a restriction on microbial growth:

$$(3) \qquad \min_{Q,T} \left\{ \frac{1}{2} hQ + \frac{FD}{Q} + G(T) \right\}$$

subject to:

$$N_0 \, e^{\mu(T)W} \leq \text{MAX}$$

In our case we can substitute $W = Q/D$ because Q/D is the maximum time an item can be in stock. MAX is a parameter to be determined in advance and has to do with guarantee on food quality and food safety.

Problem (3) can easily be reduced to (4):

$$(4) \qquad \min_{T} \left\{ \frac{1}{2} h \frac{D}{\mu(T)} \ln\left(\frac{\text{MAX}}{N_0}\right) + \frac{F\mu(T)}{\ln\left(\frac{\text{MAX}}{N_0}\right)} + G(T) \right\}$$

because (obviously) in the optimum the restriction in problem (3) is binding.

It is interesting to compare the solution of the optimization problem (3) (assuming $Q = EOQ$) with the solution of (3) without this assumption because this provides insight into the difference in costs of simultaneous control (in *Q and T*) and control based on solely $Q = EOQ$.

11.3 Determination of lot sizes and temperatures at warehouse and retail store

Now consider a more complex situation with two echelons: the warehouse (*W*) and the retail store (*R*). For the sake of simplicity we assume that we are faced with constant demand *D* at the retailer site. Now let us describe the development over time of the inventory at the retail store. In fact the picture resembles figure 1. For the sake of clarity we now use the notation Q_R instead of *Q*.

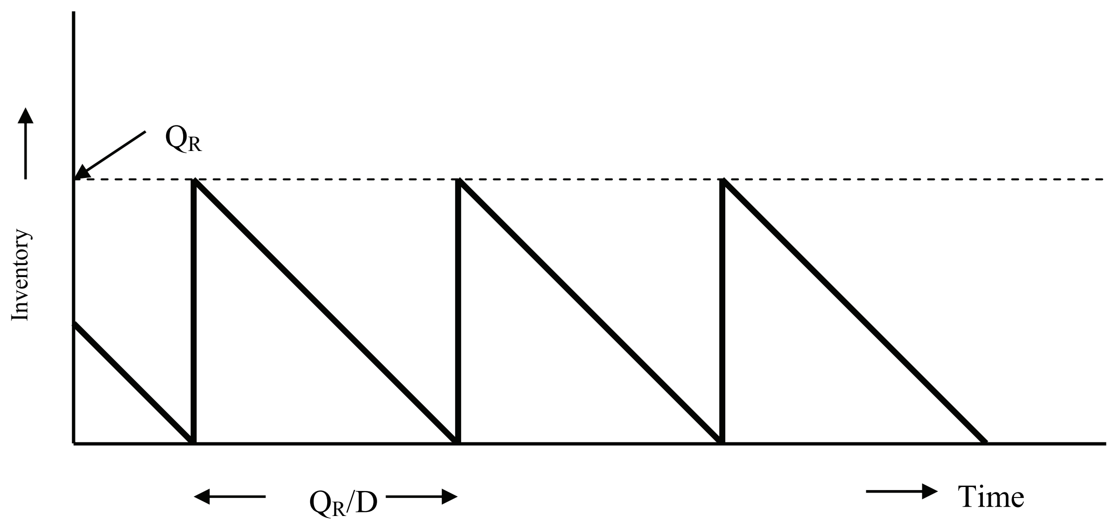

Figure 2: Inventory at the retailer store

The retailer is supplied every Q_R/D periods, at times where inventory drops to zero, with replenishment quantity Q_R. Replenishment is assumed to take place instantaneously.

These supply quantities imply a decrease of stock at the warehouse site of Q_R (see figure 3) in which, for the sake of clarity, also figure 2 has been incorporated. In figure 3 we assume, as an illustration, that $Q_W = 3\ Q_R$. The quantity Q_W is the replenishment quantity with which the warehouse is supplied. Figure 3 is a slightly modified version of figure 12.5 (see Silver et al. 1998, chapter 12).

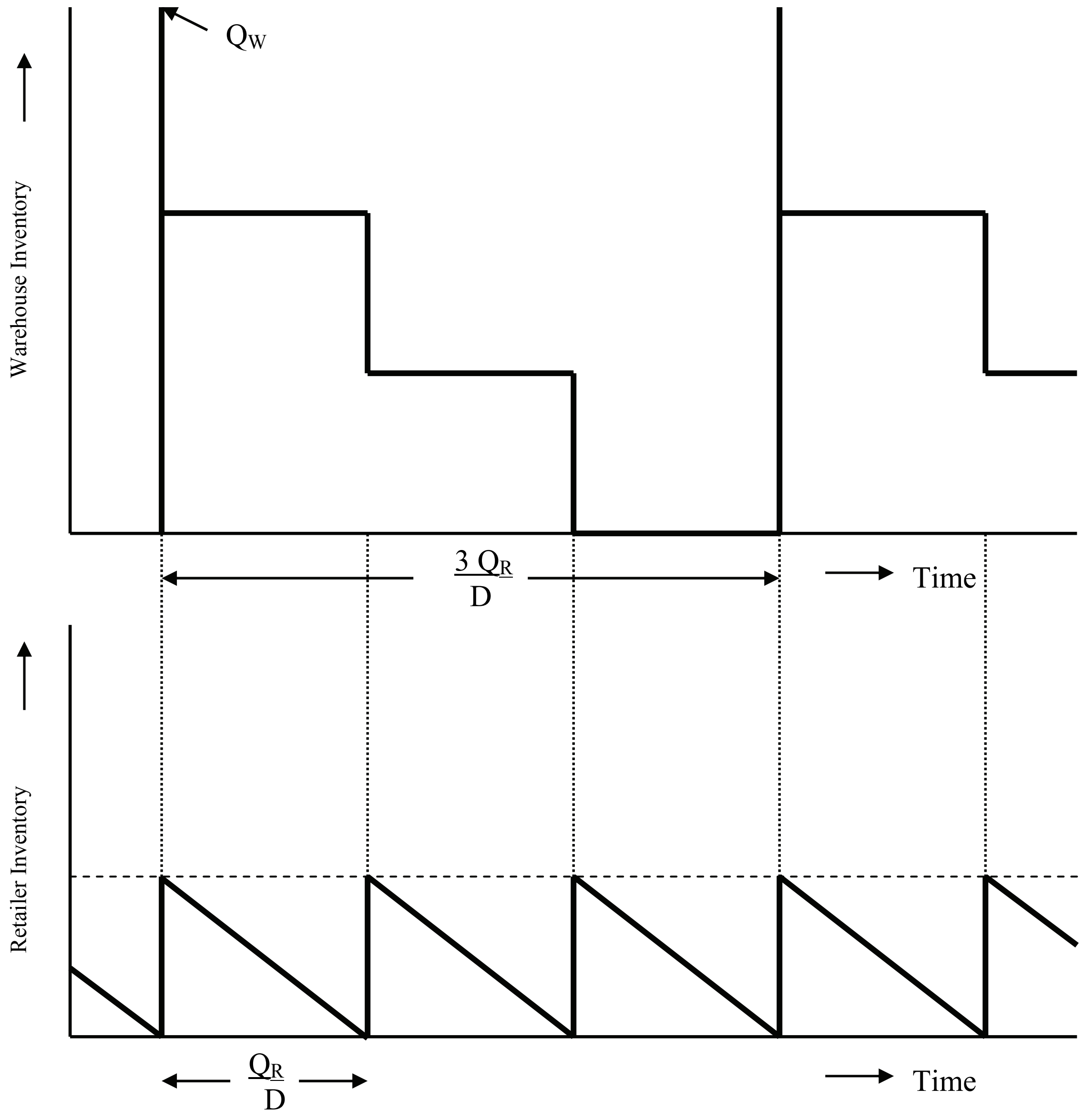

Figure 3: Inventory at the retailer store and at the warehouse site

In figure 3 we have illustrated the development over time of both inventories using the relation $Q_W = 3\ Q_R$. In the sequel we assume $Q_W = n\ Q_R$ with n = 1, 2, … This is called the "Integrality Property", for obvious reasons. In our example we have n = 3.

From figure 3 we can conclude that the maximum sojourn time for the product under consideration is:

- At the retailer site: Q_R/D periods
- At the warehouse site: $(n - 1)\ Q_R/D$ periods

If we assume that transportation time of the product from warehouse to retailer is negligible (in fact: 0), the total sojourn time of the product in the chain amounts to:

$$\frac{Q_R}{D} + (n-1)\frac{Q_R}{D} = n\frac{Q_R}{D}$$

Furthermore, if we assume different cooling regimes at the warehouse and retailer site we need the two different sojourn times $\frac{Q_R}{D}$ and $(n-1)\frac{Q_R}{D}$ because each of both sojourn times influences the growth of micro-organisms in dependence of the actual cooling regimes in each of the two echelons.

Now we formulate how to determine the optimal solution in the two-echelon case. Let h_R, h_W be the inventory cost coefficients for Retailer and Warehouse, respectively and F_R, F_W the corresponding fixed setup costs.

Now we can draw up the two-echelon optimization model:

$$(5) \qquad \min\left\{\frac{1}{2}h_R Q_R + \frac{F_R D}{Q_R} + \frac{1}{2}h_W (n-1) Q_R + \frac{F_W D}{nQ_R}\right\}$$

This expression (5) depends on Q_R and n and can be minimized easily by choosing the procedure as described in Silver et al. (1998), chapter 12.3.

As in the one-echelon situation we assume that the underlying product is perishable and its decay depends on time *and* temperature. Again we make use of formula (2) for calculating the growth of micro-organisms in each of the two echelons.

Applying this formula in each of the two echelons we arrive at the following optimization problem (now using G_R and G_W instead of G as different cost functions related to cooling):

$$(6) \qquad \min\left\{\frac{F_R D}{Q_R} + \frac{1}{2}h_R Q_R + G_R(T_R) + \frac{F_W D}{nQ_R} + \frac{1}{2}h_W (n-1) Q_R + G_W(T_W)\right\}$$

Subject to:

$$N_0 e^{\mu(T_W) Q_R (n-1)/D + \mu(T_R) Q_R / D} \leq \text{MAX}$$

Solving (6) is an optimization problem in the variables n, Q_R, T_R and T_W and can be solved easily by using standard software for non-linear programming problems.

This solution process can be simplified by concluding that in the optimal solution the restriction in (6) will be equality rather than inequality (compare formula (4)).

Solving the resulting equation gives the following expression for Q_R:

$$(7) \qquad Q_R = \frac{D}{(n-1)\mu(T_W) + \mu(T_R)} \ln\left(\frac{\text{MAX}}{N_0}\right)$$

If we substitute (7) into the object function of (6) an optimization problem in the variables n, T_W and T_R results which can be solved by minimizing the cost function (6) for different values of n.

11.4 Conclusions and Recommendations

In the foregoing we worked out a very simple two-echelon inventory model. In this model we illustrated how to balance on the one hand the cooling efforts (temperature in each of the two echelons) and on the other the reduction of sojourn time by choosing smaller lot sizes in warehouse and/or in the retail store. This illustration makes clear that for relevant choices of problem data this kind of balancing turns out to be meaningful. Finding the optimal balance has been achieved by minimization of the total relevant cost of this two-echelon system taking into account the restriction that the number of micro-organisms (per ml) never exceeds a pre-specified threshold.

Further research should focus on applying this approach in more realistic settings (variable demand, alternative products, more echelons, network in stead of chain etc.). In these settings temperature can also be replaced by more general control variables having direct influence on the quality of the relevant product.

References

Silver, E.A., Pyke, D.F., Peterson, R. (1998). Inventory Management and Production Planning and Scheduling. Third Edition. John Wiley & Sons, Inc.

Zwietering, M.H. (1993). Modeling of the microbial quality of food. Ph.D. Thesis Wageningen University, 152 pp.